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Fixed Point Results for Non-Linear Operators with Comparisons

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Abstract: The purpose of this research article is to introduce a new iteration scheme and prove convergence and stability results for it. We also claim the newly introduced iterative scheme converges faster than some of the existing iterations in the literature. Our claim is supported by numerical example.

Keywords: J-iteration, Suzuki generalized non expansive mapping, stability.

2010 AMS Subject Classification: 47H09, 47H10.

1. Introduction and Preliminaries

The theory of fixed points has become an interdisciplinary area of research as it has applications in mathematics, economics, game theory etc. In general, the solution of fixed point problem is almost impossible therefore need of iterative solution arises. Developing a faster and simpler iterative scheme to obtain the fixed point is an interesting and active area of research. Over the years different iterative schemes for finding the solution of fixed point problems for different operators have been developed by the researchers, for example, see([5,6, 8-10, 16-19])).

In 2017, Ullah and Arshad [15] introduced the M*-iteration scheme by the method

$$\begin{cases} z_n = (1-\beta_n)\,x_n + \beta_n T x_n \\ y_n = T\big((1-\alpha_n)\,T x_n + \alpha_n T z_n\big) \\ x_{n+1} = T y_n \end{cases} \quad (1.1)$$

where $\{\alpha_n\}, \{\beta_n\}$ sequences such that $\alpha_n, \beta_n \in (0,1)$.

Authors in [15], claimed that their iteration scheme is faster than existing iteration schemes in the literature such that Picard, Mann, Ishikawa, Noor, Agarwal et al. and Abbas at at.

Hussain et al. in [7], defined a new iteration scheme and named it as 'K-iteration scheme'. They proved convergence result for this iterative scheme by considering the class of Suzuki generalized non expansive mapping in uniformly convex banach space.

$$\begin{cases} z_n = (1-\beta_n)x_n + \beta_n T x_n, \\ y_n = T((1-\alpha_n)Tx_n + \alpha_n T z_n, \\ x_{n+1} = Ty_n, \end{cases} \tag{1.2}$$

Again in 2018 Ullah and Arshad [17] introduced the K*-iteration scheme by the method

$$\begin{cases} z_n = (1-\beta_n)\, x_n + \beta_n T x_n \\ y_n = T\big((1-\alpha_n)\, z_n + \alpha_n T z_n\big) \\ x_{n+1} = Ty_n \end{cases} \tag{1.3}$$

for all $n \in N$, where $\{\alpha_n\}, \{\beta_n\}$ sequences such that $\alpha_n, \beta_n \in (0,1)$.

Authors in [17] claimed that their iteration scheme if faster than the iterative process defined by [1], [2], [7-10].

Again in 2018 Ullah and Arshad [16] introduced the M-iteration scheme by the method

$$\begin{cases} z_n = (1-\alpha_n)x_n + \alpha_n T x_n \\ y_n = T z_n \\ x_{n+1} = T y_n \end{cases} \tag{1.4}$$

for all $n \in N$, where $\{\alpha_n\}$ is a sequences such that $\alpha_n \in (0,1)$. Authors claimed that this iterative scheme is even faster than the iterative schemes like Picard, Mann, Ishikawa, Noor, Abbas et al. and some other existing iteration in the literature.

In this direction Bhutia and Tiwari [5] defined the J-Iteration scheme as follows:

For some initial approximation x_0 we have

$$\begin{cases} z_n = T\big((1-\beta_n)x_n + \beta_n T x_n\big), \\ y_n = T\big((1-\alpha_n)z_n + \alpha_n T z_n\big), \\ \qquad x_{n+1} = T y_n. \end{cases} \tag{1.5}$$

Bhutia and Tiwari [5] with suitable numerical examples claimed that their iteration scheme has better rate of convergence then some of the existing iteration schemes like K-iteration, K*-iteration, M-iteration and M*-iteration scheme. Now we introduce a new iteration scheme with the relation:

For some initial approximation x_0 we have

$$\begin{cases} z_n = T\big((1-\beta_n)x_n + \beta_n T x_n\big), \\ y_n = T\big((1-\alpha_n)z_n + \alpha_n T z_n\big), \\ x_{n+1} = T((1-\gamma_n)T z_n + \gamma_n T y_n) \end{cases} \tag{1.6}$$

We claim that our newly defined iterative scheme converges faster than the iterative scheme defined by Bhutia and Tiwari [5].

Definition 1.1 [6]:-Let $\{z_n\}_{n=0}^{\infty}$ be the sequence in X. Then the iterative process $x_{n+1} = f(T, x_n)$ which converges to a fixed point q of T is said to be stable with respect to T if for $t_n = \| z_{n+1} - f(T, z_n) \|, n = 0, 1, 2, \ldots,$ we have $\lim_{n\to\infty} t_n = 0$ if and only if $\lim_{n\to\infty} z_n = q$.

Definition 1.2[4]: Let H be a non-empty subset of a Banach space X and $T: X \to X$ be a mapping. T is called a contraction mapping if there exists a real number $k < 1$such that for all $x, y \in X$ we have $d(Tx, Ty) \leq kd(x, y)$.

Definition 1.3 [15]: An operator $T: K \to K$ is said to satisfy the condition (c), if for all $x, y \in K$, we have $\frac{1}{2} d(x, Tx) \leq d(x, y)$ implies $d(Tx, Ty) \leq d(x, y)$. Any mapping satisfies condition (c) is also known as Suzuki generalized non-expansive mapping.

Proposition1.4 [15]: Let H be a non-empty subset of a Banach space X and $T: X \to X$ be a mapping. Then

1. If T is non-expansive, then T is Suzuki generalized non-expansive mapping.
2. If T is Suzuki generalized non-expansive mapping and has a fixed point, then T is a quasi-non expansive mapping.

Lemma 1.5[14]: Let H be a non-empty subset of a Banach space X and $T: X \to X$ be a Suzuki generalized non expansive mapping. Then for all $x, y \in X$, we have

$$\| Tx - Ty \| \leq 3 \| Tx - x \| + \| x - y \| \qquad (1.7)$$

Lemma 1.6 [13]: Suppose X is a uniformly convex Banach space and $\{s_n\}$ be any sequence of real numbers such that $0 < s_n < 1$ for all $n \geq 1$. Let $\{a_n\}$ and $\{b_n\}$ be any two sequences of real numbers in X such that $\limsup_{n\to\infty} \| a_n \| \leq r$, $\limsup_{n\to\infty} \| b_n \| \leq r$ and $\limsup_{n\to\infty} \| s_n a_n + (1 - s_n) b_n \| = r$ holds for some non-negative constant r. Then $\lim_{n\to\infty} \| a_n - b_n \| = 0$.

Remark 1.7[14]: Let H be a non-empty subset of a Banach space X and let $\{x_n\}_{n=0}^{\infty}$ be a bounded sequence in X. For $x \in X$, we set $r(x, \{x_n\}) = \limsup_{n\to\infty} \| x_n - x \|$. The asymptotic radius of $\{x_n\}$ relative to H is given by $r(H, \{x_n\}) = \inf\{r(x, \{x_n\}): x \in H\}$ and the asymptotic center of $\{x_n\}$ relative to H is the set

$$A(H, \{x_n\}) = \{x \in H: r(x, \{x_n\}) = r(H, \{x_n\})\}.$$

2. Main Results

Theorem 2.1: Let H be a non-empty subset of a Banach space X and $T: X \to X$ be a contraction mapping. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence of defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}, \{\gamma_n\}_{n=0}^{\infty}$ in $[0, 1]$ satisfying $\sum_{n=0}^{\infty} \gamma_n = \infty$. Then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to unique fixed point of T.

Proof: Since T is a contraction mapping so by Banach contraction principle it has a unique fixed point. Let q be the unique fixed point of T. Now by the iteration scheme (1.6) we have,

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$$\| z_n - q \| = \| T((1-\beta_n)x_n + \beta_n T x_n) - q \|$$

$$\leq k \| (1-\beta_n)x_n + \beta_n T x_n - q \|$$

$$\leq k[(1-\beta_n) \| x_n - q \| + \| \beta_n T x_n - q \|]$$

$$\leq k[(1-\beta_n) \| x_n - q \| + \beta_n k \| \beta_n x_n - q \|]$$

$$\leq k[1-\beta_n(1-k)] \| x_n - q \|$$

By the hypothesis of theorem, we have $1-\beta_n(1-k) < 1$ so we can write

$$\| z_n - q \| \leq k \| x_n - q \| \qquad (2.1)$$

And

$$\| y_n - q \| = \| T((1-\alpha_n)z_n + \alpha_n T z_n) - q \|$$

$$\leq k \| (1-\alpha_n)z_n + \alpha_n T z_n - q \|$$

$$\leq k[(1-\alpha_n) \| z_n - q \| + \alpha_n \| T z_n - q \|]$$

$$\leq k[(1-\alpha_n) \| z_n - q \| + \alpha_n k \| z_n - q \|]$$

$$\leq k[1-\alpha_n(1-k)] \| z_n - q \|$$

Again, by the hypothesis of theorem we have $1-\alpha_n(1-k) < 1$ and using (2.1) we have

$$\| y_n - q \| \leq k \| z_n - q \|$$

$$\leq k^2 \| x_n - q \| \qquad (2.2)$$

And by using (1.6) and (2.1) and (2.2) we have

$$\| x_{n+1} - q \| = \| T((1-\gamma_n)T z_n + \gamma_n T y_n) - q \|$$

$$\leq k \| (1-\gamma_n)T z_n + \gamma_n T y_n - q \|$$

$$\leq k[(1-\gamma_n) \| T z_n - q \| + \gamma_n \| T y_n - q \|]$$

$$\leq k[(1-\gamma_n)k \parallel z_n - q \parallel + \gamma_n k \parallel y_n - q \parallel]$$

$$\leq k[(1-\gamma_n)k^2 \parallel x_n - q \parallel + \gamma_n k^3 \parallel x_n - q \parallel]$$

$$\leq k^3[1-\gamma_n(1-k)] \parallel x_n - q \parallel \qquad (2.3)$$

By repeating the above arguments, we have

$$\parallel x_n - q \parallel \leq k^3[1-\gamma_{n-1}(1-k)] \parallel x_{n-1} - q \parallel$$

$$\parallel x_{n-1} - q \parallel \leq k^3[1-\gamma_{n-2}(1-k)] \parallel x_{n-2} - q \parallel$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$\parallel x_1 - q \parallel \leq k^3[1-\gamma_0(1-k)] \parallel x_0 - q \parallel$$

Combining all the above inequalities we have

$$\parallel x_{n+1} - q \parallel \leq k^{3(n+1)} \parallel x_0 - q \parallel \prod_{i=0}^{n} 1-\gamma_i(1-k)$$

Now $k < 1$ so $1-k > 0$ and $\gamma_i \leq 1$ for all $n \in N$, hence we have $1-\gamma_i(1-k) < 1$. We know that $1 - x \leq e^{-x}$ for all $x \in [0,1]$. So we have

$$\parallel x_{n+1} - q \parallel \leq k^{3(n+1)} \parallel x_0 - q \parallel e^{-(1-k)\sum_{i=0}^{n}\gamma_i} \qquad (2.4)$$

Taking limit $n \to \infty$ both sides we have

$\lim_{n\to\infty} \parallel x_n - q \parallel = 0$. This completes the proof.

Theorem 2.2: Let H be a non-empty subset of a Banach space X and $T: X \to X$ be a contraction mapping. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence of defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}, \{\gamma_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfying $\sum_{n=0}^{\infty}\gamma_n = \infty$. Then the sequence $\{x_n\}_{n=0}^{\infty}$ is T- stable.

Proof: Let $\{x_n\}_{n=0}^{\infty}$ be any sequence in H and let the sequence generated by (1.6) be $t_{n+1} = f(T, x_n)$ and let it converges to the unique fixed point q of T. Suppose $\delta_n = \| t_{n+1} - f(T, t_n) \|$. Now we will prove that $\lim_{n\to\infty} \delta_n = 0$ if and only if $\lim_{n\to\infty} t_n = q$. First suppose that $\lim_{n\to\infty} t_n = q$. Then we have

$$\delta_n = \| t_{n+1} - f(T, t_n) \|$$

$$\leq \| t_{n+1} - q \| + \| f(T, t_n) - q \|$$

$$\leq \| t_{n+1} - q \| + k^3 [1 - \gamma_n (1 - k)] \ \| t_n - q \|$$

Taking limit $n \to \infty$ both sides of the above inequality we have $\lim_{n\to\infty} \delta_n = 0$.

Conversely suppose that $\lim_{n\to\infty} \delta_n = 0$. Now we have

$$\| t_{n+1} - q \| \leq \| t_{n+1} - f(T, t_n) \| + \| f(T, t_n) - q \|$$

$$\leq \ \delta_n + \| f(T, t_n) - q \|$$

Using theorem (2.1) we can write

$$\| t_{n+1} - q \| \leq \delta_n + \ [1 - \gamma_n (1 - k)] \ \| t_n - q \|$$

Now $0 < k < 1$ and $\gamma_i \leq 1$ for all $n \in N$ and $\lim_{n\to\infty} \delta_n = 0$. Then from the above inequality and lemma (1.6) we have, $\lim_{n\to\infty} \| t_n - q \| = 0$. Hence the sequence $\{x_n\}_{n=0}^{\infty}$ is T-stable.

Now we establish some fixed point results related to Suzuki generalized non-expansive mapping.

Lemma 2.3: Let H be a non-empty closed convex subset of a Banach space X and $T: X \to X$ be a Suzuki generalized non-expansive mapping with $F(T) \neq \emptyset$. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence of defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^{\infty}$,

$\{\beta_n\}_{n=0}^{\infty}, \{\gamma_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfying $\sum_{n=0}^{\infty} \gamma_n = \infty$. Then $\lim_{n\to\infty} \| x_n - q \|$ exists for all $q \in F(T)$.

Proof: Let $q \in F(T)$. Now the convexity of H we have, $(1-\gamma_n)x_n + \gamma_n T x_n \in H$ for all $n \in N$. Since T is Suzuki generalized non-expansive mapping so we can write

$$\frac{1}{2} \| q - Tq \| = 0 \leq \| q - ((1-\gamma_n)x_n + \gamma_n T x_n) \|$$

which implies that

$$\| Tq - T\big((1-\gamma_n)x_n + \gamma_n T x_n\big) \| \leq \| q - ((1-\gamma_n)x_n + \gamma_n T x_n) \|$$

Now from the iterative process (1.6) we have

$$\begin{aligned}
\| z_n - q \| &= \| T((1-\beta_n)x_n + \beta_n T x_n) - Tq \| \\
&\leq \| ((1-\beta_n)x_n + \beta_n T x_n) - q \| \\
&\leq (1-\beta_n) \| x_n - q \| + \beta_n \| T x_n - q \| \\
&\leq (1-\beta_n) \| x_n - q \| + \beta_n \| x_n - q \| \\
&\leq \| x_n - q \| \qquad (2.5)
\end{aligned}$$

Now

$$\begin{aligned}
\| y_n - q \| &= \| T((1-\alpha_n)z_n + \alpha_n T z_n) - Tq \| \\
&\leq \| (1-\alpha_n)z_n + \alpha_n T z_n - q \| \\
&\leq (1-\alpha_n) \| z_n - q \| + \alpha_n \| T z_n - q \| \\
&\leq (1-\alpha_n) \| z_n - q \| + \alpha_n \| z_n - q \| \\
&\leq \| z_n - q \| \qquad (2.6) \\
&\leq \| x_n - q \| \qquad (2.7)
\end{aligned}$$

Again using (1.6), (2.5) and (2.7),

$$\| x_{n+1} - q \| = \| T((1-\gamma_n)Tz_n + \gamma_n Ty_n) - Tq \|$$

$$\leq \| (1-\gamma_n)Tz_n + \gamma_n Ty_n - q \|$$

$$\leq (1-\gamma_n) \| Tz_n - q \| + \gamma_n \| Ty_n - q \|$$

$$\leq (1-\gamma_n) \| z_n - q \| + \gamma_n \| y_n - q \|$$

$$\leq (1-\gamma_n) \| x_n - q \| + \gamma_n \| x_n - q \|$$

$$\leq \| x_n - q \|$$

Hence $\{\| x_n - q \|\}$ is bounded and non-increasing for all $q \in F(T)$. Hence $\lim_{n\to\infty} \| x_n - q \|$ exists for all $q \in F(T)$.

Theorem 2.4: Let H be a non-empty closed convex subset of a Banach space X and $T: X \to X$ be a Suzuki generalized non-expansive mapping. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence of defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^{\infty}, \{\beta_n\}_{n=0}^{\infty}, \{\gamma_n\}_{n=0}^{\infty}$ in $[0,1]$ satisfying $\sum_{n=0}^{\infty} \gamma_n = \infty$. Then $F(T) \neq \emptyset$ if and only if $\{x_n\}_{n=0}^{\infty}$ is bounded and $\lim_{n\to\infty} \| Tx_n - x_n \| = 0$.

Proof: First suppose that $F(T) \neq \emptyset$. Let $q \in F(T)$. Then by lemma (2.4), $\lim_{n\to\infty} \| x_n - q \|$ exists for all $q \in F(T)$ and $\{x_n\}_{n=0}^{\infty}$ is a bounded sequence.

Let $\lim_{n\to\infty} \| x_n - q \| = \theta$ for some $\theta > 0$. Now from (2.5) we have

$$\limsup_{n\to\infty} \| z_n - q \| \leq \limsup_{n\to\infty} \| x_n - q \| = \theta$$

By proposition (1.4), we have

$$\limsup_{n\to\infty} \| Tx_n - q \| \leq \limsup_{n\to\infty} \| x_n - q \| = \theta$$

Now using (1.6) and (2.5) we have

$$\| x_{n+1} - q \| = \| T((1-\gamma_n)Tz_n + \gamma_n T y_n) - Tq \|$$

$$\leq \| (1-\gamma_n)Tz_n + \gamma_n T y_n - q \|$$

$$\leq (1-\gamma_n) \| Tz_n - q \| + \gamma_n \| Ty_n - q \|$$

$$\leq (1-\gamma_n) \| z_n - q \| + \gamma_n \| y_n - q \|$$

$$\leq (1-\gamma_n) \| z_n - q \| + \gamma_n \| z_n - q \|$$

$$\leq \| z_n - q \|$$

Which implies that $\| x_{n+1} - q \| \leq \| z_n - q \|$ and hence

$$\theta \leq \liminf_{n\to\infty} \| z_n - q \|$$

Therefore we can write

$$\theta \leq \liminf_{n\to\infty} \| z_n - q \| \leq \limsup_{n\to\infty} \| z_n - q \| \leq \theta$$

Thus we obtain $\lim_{n\to\infty} \| z_n - q \| = \theta$.

Now

$$\theta = \lim_{n\to\infty} \| T((1-\beta_n)x_n + \beta_n T x_n) - q \|$$

$$\leq \lim_{n\to\infty} \| (1-\beta_n)x_n + \beta_n T x_n - q \|$$

$$\leq \lim_{n\to\infty} [(1-\beta_n) \| x_n - q \| + \beta_n \| Tx_n - q \|]$$

$$\leq \lim_{n\to\infty} (1-\beta_n) \| x_n - q \| + \beta_n \| x_n - q \|$$

$$\leq \lim_{n\to\infty} \| x_n - q \| \leq \theta.$$

Hence we can write

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$$\theta \leq \lim_{n\to\infty} \| (1-\beta_n)(x_n - q) + \beta_n(Tx_n - q) \| \leq \theta.$$

Thus $\lim_{n\to\infty} \| (1-\beta_n)(x_n - q) + \beta_n(Tx_n - q) \| = \theta$.

Using lemma (1.6) and the above calculations we have $\lim_{n\to\infty} \| Tx_n - x_n \| = 0$.

Conversely let $\{x_n\}_{n=0}^{\infty}$ is bounded and $\lim_{n\to\infty} \| Tx_n - x_n \| = 0$. Let $q \in A(H, \{x_n\})$. Now by lemma 1.5, we have

$$\begin{aligned} r(Tq, \{x_n\}) &= \limsup_{n\to\infty} \| x_n - Tq \| \\ &\leq \limsup_{n\to\infty} (3 \| Tx_n - x_n \| + \| x_n - q \|) \\ &\leq \limsup_{n\to\infty} (\| x_n - q \|) \end{aligned}$$

which implies that $Tq \in A(H, \{x_n\})$. Since X is uniformly convex Banach space. It follows that $A(H, \{x_n\})$ is singleton. Hence $Tq = q$ implies that $q \in F(T)$ and hence $F(T) \neq \emptyset$. This completes the proof.

Bhutia and Tiwari [5] claimed by giving example that their J-iteration scheme has much better rate of convergence than M-iteration, M*-iteration, K-Iteration and K*-iteration. We now compare the rate of convergence of our iterative scheme with J-iteration scheme by considering the example of Bhutia and Tiwari [5].

Example 2.3: Consider the mapping$T(x) = (x+2)^{\frac{1}{2}}$. Clearly T is a contraction mapping and $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$ be the sequence defined by $\alpha_n = \beta_n = \gamma_n = \frac{1}{4}$ for all $n \in N$.

Iteration	J-Iteration	Iteration (1.6)
0	4	4
1	2.0182876356079	2.01797953158
2	2.0001883967869	2.00018255043
4	2.0000000200447	2.00000001887
5	2.0000000002067	2.00000000019
6	2.0000000000021	**2**
7	2.0000000000002	**2**
8	**2**	**2**

Table 2.1

Table 2.1, clearly indicate that the iterative scheme (1.6) has higher rate of convergence than the J-iteration process.

References:

1. Abbas, M. and Nazir, T. (2014). A new faster iteration process applied to constrained minimization and feasibility problems. Mater. Vesn. 66, 223-234.
2. Agarwal, R. P., O'Regan, D. and Sahu, D. R. (2007). Iterative construction of fixed points of nearly asymptotically non-expansive mappings. J. Nonlinear Convex Anal., 8, 61-79.
3. Ali, J., Ali, F. and Kumar, P. (2019). Approximation of fixed points for Suzuki's generalized nonexpansive mappings, Mathemaics, 1-11.
4. Berinde, V. (2007). Iterative Approximation of Fixed Points, Springer, Berlin.
5. Bhutia,J.D. and Tiwari, K. (2019). New iteration process for approximating fixed points in Banach spaces. Journalof linear and toplogical algebra. 8(4), 237-250.
6. Erturk, M. and Gursoy, F. (2019). Some convergence, stability and data dependency results for Picard-S iteration method of quasi-strictly contractive operators, Mathematica Bohemica, 144, 69-83.
7. Harder, A.M. (1987). Fixed point theory and stability results for fixed point iteration procedure, University of Missouri-Rolla, Missouri.
8. Hussain, N., Ullah, K. and Arshad, M. (2018). Fixed point approximation for Suzuki generalized nonexpansive mappings via new iteration process, J. Nonlinear Convex Anal. 19 (8),1383-1393.
9. Thakur, B.S., Thakur, D. and Postolache, M. (2016). A new iterative scheme for numerical reckoning fixed points of Suzuki's generalized nonexpansive mappings. Appl. Math. Comp., 275, 147-155.
10. Thakur, B.S., Thakur, D. and Postolache, M. (2016). A new iterative scheme for approximating fixed points of non-expansive mappings. Filomat. 30, 2711-2720.
11. Mann, W.R. (1953). Mean value methods in iteration, Proc. Am. Math. Soc., 4, 506-510.

12. Noor, M. A. (2000). New approximation schemes for general variational inequalities. Journal of Mathematical Analysis and Applications, 251, 217–229.
13. Schu, J. (1991). Weak and strong convergence to fixed points of asymptotically non expansive mappings. Bull. Aust. Math. Soc. 43, 153-159.
14. Soltuz, S.M. and Grosan, T. (2008). Data dependence for Ishikawa iteration when dealing with contractive like operators. Fixed point theory and applications.
15. Suzuki, T. (2008). Fixed point theorems and convergence theorems for some generalized non-expansive mappings, J.Math. Anal. 340 (2), 1088-1095.
16. Ullah, K. and Arshad, M. (2016). On different results for new three step iteration process in Banach Spaces. Springer Plus, 2016, 1-15.
17. Ullah, K. and Arshad, M. (2017). New iteration process and numerical reckoning fixed point in Banach space. U.P.B. sci. bull. (Series A). 79(4), 113-122.
18. Ullah, K. and Arshad, M. (2018). Numerical reckoning fixed point for Suzuki's generalized non-expansive mappings via new iterative process. Filomat. 32, 187-196.
19. Ullah, K. and Arshad, M. (2018). New three-step iteration process and fixed point approximation in Banach Spaces. Journal of linear and topological algebra, 7(2), 87-100.